Research statement

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A link is a disjoint union of closed curves embedded in $\mathbb{R}^3$. Links that can be transformed into one another through an ambient isotopy in $\mathbb{R}^3$ are considered the same. A surface-link is a disjoint union of closed surfaces embedded in $\mathbb{R}^4$, representing a higher-dimensional generalization of links. Similar to links, surface-links that can be transformed into one another through an ambient isotopy in the space are considered the same.

I have conducted research on the classification problem of surface-links. In particular, the motivation of my research is the question of how to describe and visualize surface-links embedded in $\mathbb{R}^4$, which cannot be directly observed.

Development of plat form presentations for surface-links

Links admit representations using braids, which enable algebraic and combinatorial discussions of links. As a higher-dimensional generalization of braids, there exist the concept of braided surfaces and 2-dimensional braids. Previous research provided a representation of surface-links using 2-dimensional braids (closed braid representation). However, this representation could only describe orientable surface-links. I developed a new representation method using braided surfaces (plat form presentation) and demonstrated that it can represent all surface-links, regardless of orientability.

Characterization of the knot group of surface-links

The knot group is the fundamental group of the complement of a surface-link and has long been studied as a fundamental invariant in Knot theory. By utilizing a plat form presentation of surface-links, I introduced a method to compute the group presentation of the knot group. In particular, by applying the algebraic properties of braided surfaces, I provided an algebraic characterization—necessary and sufficient conditions —for a given group to be the knot group of a surface-link.

Introduction and application of knitted surfaces

This research is a collaboration with Inasa Nakamura (Saga University). The focus is on properly embedded compact surfaces with boundary in B^4. braided surfaces are compact surfaces with boundary in B^4. However, there exist such surfaces that are not ambient isotopic to braided surfaces. Therefore, We introduced knitted surfaces as a generalization of braided surfaces and showed that all compact surfaces with boundary in B^4 are ambient isotopic to some knitted surface. Furthermore, by taking the closure, all surface-links can be represented as closed braids, regardless of orientability.