Future research plans

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In the spacetime of Kerr black hole, there exists a hidden symmetry described by a special tensor field called the Conformal Killing-Yano (CKY) tensor. Such symmetry is believed to play an important role not only in the variable separation appearing in perturbation equations of black hole spacetimes but also in the analysis of (in)stability of spacetime. Historically, this symmetry was discovered by Walker and Penrose (1970) as a symmetry of 4-dimensional Kerr black hole. Furthermore, the introduction of CKY from a purely mathematical perspective dates back to the studies of geometers such as Kashiwada (1968) and Tachibana (1969). In the latter half of the 20th century, the interest in unified theories including gravity, such as superstring theory and supergravity theory, motivated the study of higher-dimensional black hole spacetimes. We have shown that CKY can be extended to higher-dimensional spacetimes [45]-[69]. The key result is the proof that the only spacetime allowing CKY is the Kerr-NUT-(Anti) de Sitter black hole spacetime [51][52].

Applying such research to geometry, it is possible to induce Einstein metrics on compact manifolds using the analytical continuation of black hole spacetimes allowing CKY. The Einstein metric obtained in joint research with Hashimoto-Sakaguchi [35] appears very natural when viewed through CKY. There seems to be a deep connection between Lorentzian geometry and Riemannian geometry. Page metric, toric Sasaki-Einstein metrics, and Einstein metrics on spherical bundles with torus action can also be constructed from higher-dimensional black hole spacetimes. An overview of these results has already been published in review papers [59] and [72][73], and further developments are expected through a deeper geometric or physical understanding.

In recent years, black hole research has made tremendous progress, including the discovery of gravitational waves (2015) and the imaging of the supermassive black hole at the center of a galaxy (2019). These advancements have significantly shifted the perception of black holes from being purely mathematical solutions to Einstein's equations to being real, observable entities. In such a period, it is valuable to revisit classical theoretical research on black holes, incorporating modern methods. This study attempts to reconstruct the operators known as "Wald's quartet," proposed by Wald in 1978, from the perspective of CKY in the Kerr black hole spacetime. Wald's quartet provides a geometric interpretation of the separability of variables in the gravitational perturbation equation (Teukolsky equation) and is expected to play an important role in the analysis of gravitational waves.

The effects of magnetohydrodynamics on black hole accretion disks and jets have been observed through X-ray observational data and numerical simulations. This study considers the black hole spacetime with a perfect fluid as a model to understand such phenomena. By incorporating the degrees of freedom of the fluid as torsion into CKY, the Einstein equations are transformed into the equations of harmonic maps. The perfect fluid undergoing rigid body rotation around the black hole (Wahlquist solution) discussed in papers [62][68] can also be derived using this method. This study aims to clarify the geometric correspondence between the Einstein equations and harmonic maps and to construct black hole spacetimes that include rotationg perfect fluids.