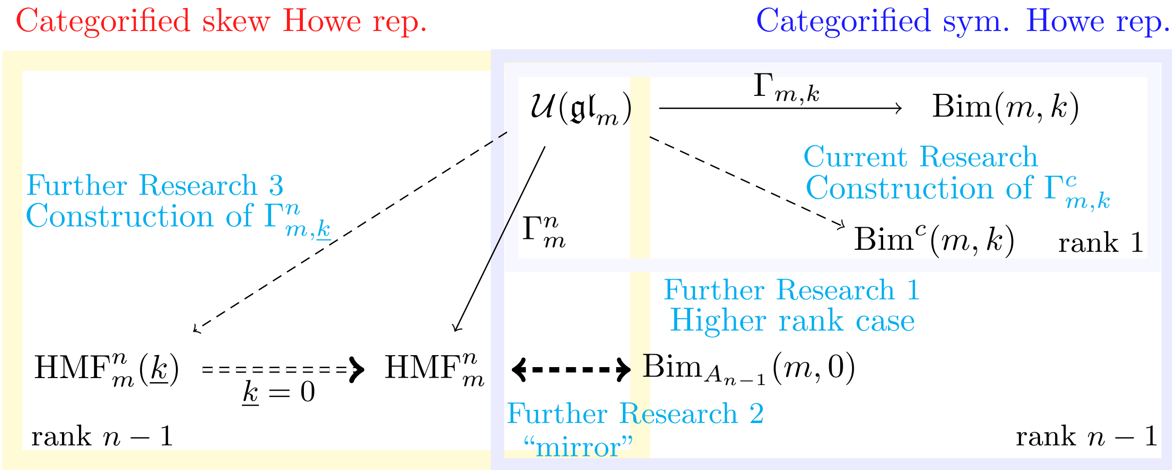

Solid arrows in the figure represent results obtained in my previous works, while dashed arrows indicate future areas of study.

In my previous works, general type deformed Webster algebras $W^{\mathfrak{g}}$ are defined as subalgebras of the Khovanov-Lauda-Rouquier algebra. Since the Khovanov-Lauda-Rouquier algebra has a p-DG structure, this structure is also considered for the algebra $W^{\mathfrak{g}}$. Based on the general deformed Webster algebra $W^{\mathfrak{g}}$, I will work on constructing homological link invariants that refine the quantum invariants of links obtained from quantum groups $U_q(\mathfrak{g})$ and their representations, as well as constructing homological invariants of three-dimensional manifolds.

Research Plan

This research will be conducted in accordance with the plan of the Grant-in-Aid for Scientific Research/Area No.:22K03318.

In the academic year 2025, I will advance my research based on the discussions held when Khovanov stayed in Japan in 2025 (scheduled for 4 days in February 2025). Additionally, I plan to discuss the following topics with Khovanov, Lauda, Qi, and Sussan in the United States.

(1) On the symmetric product $S^k(\mathbb{C}^n \otimes \mathbb{C}^m)$, we have a left $U_q(\mathfrak{sl}_n)$ action and a right $U_q(\mathfrak{gl}_m)$ action such that these actions commute. Thus, we have the representation

$$\gamma_m^{\mathfrak{sl}_n} : U_q(\mathfrak{gl}_m) \to \bigoplus_{\sum_{\alpha=1}^m i_\alpha = k, \sum_{\alpha=1}^m j_\alpha = k} \mathrm{Hom}_{U_q(\mathfrak{sl}_n)}(S^{i_1} \otimes \cdots \otimes S^{i_m}, S^{j_1} \otimes \cdots \otimes S^{j_m}).$$

It is expected that there exists a categorification of this representation on a bimodule category of the deformed Webster algebra $W^{A_{n-1}}$. I will continue to work on this challenge this year.

(2) We can introduce the p-DG structure on the algebra $W^{\mathfrak{g}}$ defined as a subalgebra of the Khovanov-Lauda-Rouquier algebra since these algebras have a p-DG structure. Using the p-DG structure, I will work on a categorification of representations of quantum groups at roots of unity.

(3) The above (1) and (2) are studies on the categorification of structures appearing in symmetric tensor products. I expect that a similar categorification can be constructed in the case of anti-symmetric tensor products using the category of matrix factorizations HMF.