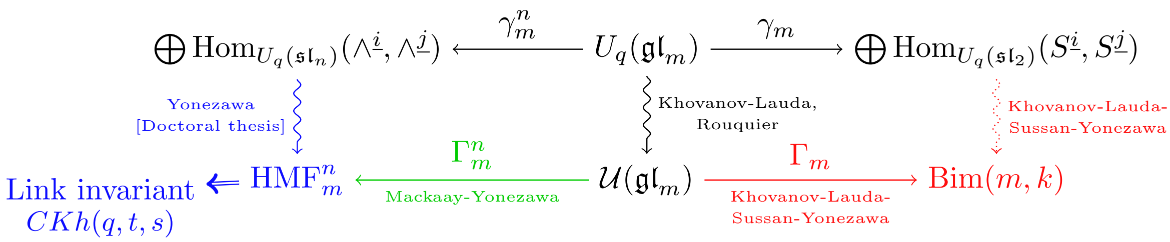
M. Khovanov defined a homological link invariant that refines the Jones polynomial. The Jones polynomial is a quantum link invariant constructed using the quantum group $U_q(sl_2)$ and its two-dimensional irreducible representation. Based on this, I have been working on solving the following problem:

Can we construct homological link invariants that refine other quantum link invariants?

Quantum link invariants, consisting of quantum groups at a root of unity, can be extended to invariants of three-dimensional manifolds. I have also been working on the following question:

Can we construct homological invariants of 3-manifolds?



(1) Summary of the paper "Quantum $(\mathfrak{sl}_n, \wedge V_n)$ link invariant and matrix factorizations": Khovanov and Rozansky constructed homological invariants that refine the quantum link invariants obtained from the quantum group $U_q(\mathfrak{sl}_n)$ and its n-dimensional irreducible representation. In this paper, we generalize Khovanov–Rozansky's theory and define a link invariant $CKh(q,t,s)$, which is a refinement of the link invariant $CJ_n(q)$ derived from $U_q(\mathfrak{sl}_n)$ and its fundamental representations (study in blue in the above figure). Note that $CJ_n(q) = CKh(q,-1,1)$.

(2) Summary of the paper "$\mathfrak{sl}_N$-Web categories and categorified skew Howe duality": We constructed a functor $\Gamma_m^n : \mathcal{U}(\mathfrak{gl}_m) \to \mathrm{HMF}_m^n$, where $\mathcal{U}(\mathfrak{gl}_m)$ is a categorification of the quantum group $U_q(\mathfrak{gl}_m)$ and HMF_n^m is a category of matrix factorizations (study in green in the above figure). Using this functor, we can construct homological invariants that refine the link invariants obtained from $U_q(\mathfrak{sl}_n)$ and its fundamental representations through an action of the braid group on the category $\mathcal{U}(\mathfrak{gl}_m)$.

(3) Summary of the paper "Braid group actions from categorical symmetric Howe duality on deformed Webster algebras": We defined a deformed Webster algebra $W(s,k)$ and constructed a functor Γ_m from $\mathcal{U}(\mathfrak{gl}_m)$ to the bimodule category $\mathrm{Bim}(m,k)$ of $W(\mathbf{s},k)$ (study in orange in the above figure). Using this functor, we constructed a braid group action on the bimodule category $\mathrm{Bim}(m,k)$.

(4) Summary of the paper "A braid group action on a p-DG homotopy category": Khovanov proposed an idea of categorifying a root of unity by introducing a p-DG structure on a category. In this paper, we define a p-DG structure on the category $\mathrm{Bim}(m,k)$ and construct a braid group action on $\mathrm{Bim}(m,k)$ that is consistent with the p-DG structure.