Research plan

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In 2003, G. Perelman proved Thurston's Geometrization Conjecture. It was proved that 3-dimensional manifolds have a unique geometric structure. Hyperbolic structures are the most important among these geometric structures. It is also useful to consider hyperbolic structures when studying the properties of 3-dimensional manifolds.

If two hyperbolic 3-manifolds or orbifolds M_1, M_2 have homeomorphic finite-sheeted covering spaces, we say M_1 and M_2 are **commensurable**. "commensurability" is an equivalence relation.

The following problem is considered.

Problem 1 *For given two hyperbolic manifolds (orbifolds), determine whether these are commensurable or not.*

If two hyperbolic manifolds (orbifolds) are "arithmetic", this problem is solved easily. We can assume these two hyperbolic manifolds (orbifolds) are non-arithmetic.

For non-arithmetic hyperbolic 3-manifold M, there exists an orbifold $C(M)$, which is called commensurator orbifold of M. It is proved that $C(M)$ has the minimal volume among the commensurability class of M. Comensurator is a **complete commensurability class invariant**.

For a cusped hyperbolic manifold, we can determine the commensurator of it by using Epstein-Penner decomposition. Thus, Problem 1 can be rewritten as follows..

Problem 2 *For a given compact non-arithmetic hyperbolic 3-manifold M, determine the commensurator of it.*

I have partially solved this problem. I decided the commensurators of orbifolds corresponds to the Coxeter groups of prisms and Löbell polyhedra. It is expected commensurators can be obtained for other Coxeter groups in the same way. I will generalize this method for compact hyperbolic manifolds. I want to aim for a complete solution to Problem 2

The smallest and second smallest hyperbolic 3-orbifolds of volume have been determined. These orbifolds are arithmetic. It is not known the smallest non-arithmetic hyperbolic 3-orbifold of volume. The commensurator orbifolds have small volumes. By calculating many examples, it seems to be useful for determining the minimal volume of non-arithmetic hyperbolic 3-orbfold.

Johnson, Kellerhals, Ratcliffe study $n(\geq 4)-$dimensional Coxeter groups. We will calculate the commensurators of $n(\geq 4)-$dimensional Coxeter groups in the same way as in the three-dimensional case