Research Result

Reiji Yoshioka

• 2d/4d(5d) correspondence and irregular limit

The 2d/4d correspondence states the equivalence between the conformal block in 2d conformal field theory and the instanton partition function in 4d $su(n)$ supersymmetric gauge theory.

On the gauge theory side, the number of matters is $N_f = 2n$ in the original 2d/4d correspondence. By taking the mass infinity limit, the degrees of freedom of the matters are decoupled and the theory goes to that with $N_f < 2n$. On the CFT side, the counterpart is known as an irregular conformal block. The β-deformed matrix model, which is equivalent to the conformal block, converts to a unitary matrix model with log-type potentials in the irregular limit. Previous studies have been mostly concerned with $n = 1$ and little attention has been paid to the irregular limit in general n, i.e., multi-matrix model in the irregular limit. I have established the irrgular limit procedure for $N_f = 2n-1, 2n-2$. In the $N_f = 2n-2$ model, I obtained the mass relation for the matter field that achieves maximum discrete symmetry based on the Dynkin diagram of the gauge group. In the parameter spaces given by this relation, the corresponding Seiberg-Witten curves are shown to be maximally degenerate.

In the above mentioned unitary matrix model with log-type potential ($n = 2$, $N_f = 2$, $\beta = 1$), the Painlevé II equation appears as a string equation at the appropriate double-scale limit. The solution of this equation is related to the free energy of the theory. I investigated the behavior of its nonperturbed part. It can also be read off by a direct calculation of the matrix model, by examining the work done against the barrier of the effective potential by a single eigenvalue lifted from the sea of the filled ones. The results were shown to be consistent.

• Unitary matrix model and AD theory

The supersymmetric gauge theories at the low energy include those that cannot be obtained from the construction of original theory, such as Argyres-Douglas (AD) theories. I considered the extension of the Gross-Witten-Wadia (GWW) model, the well-known unitary matrix model, to a model with two coupling constants. In the unitary matrix model, the eigenvalues are distributed on circle, and the distribution (called phase) depends on the coupling constants. The unitary matrix model at the phase transition point corresponds to the AD theories, and the investigation of the phase structure is interesting. I studied the phase diagram of the extended GWW model and succeeded in determining it. The phase diagram is depicted in Figure 1, consisting of three distinct phases and one triple point. The GWW model exists on the vertical axis λ, and the addition of the horizontal axis τ leads to the emergence of new phase (called 2-gap). I also computed the free energy and identified the corresponding AD theory on each phase transition line. In particular, on the red line in Figure 1, a type of AD theory appears that is not present in the original GWW model.

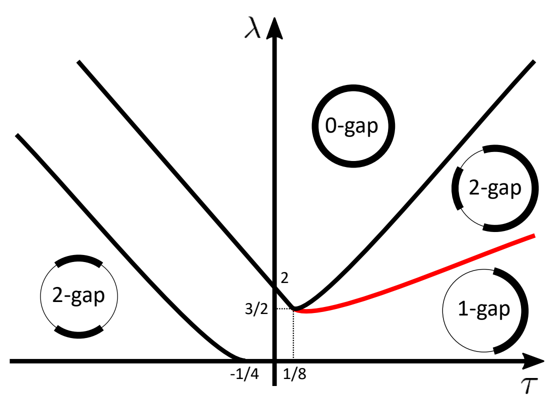

Figure 1: Phase diagram. Each phase can be classified by the number of regions without eigenvalues (gap) and is named 0-gap, 1-gap, and 2-gap.The circular diagram surrounding each name represents the eigenvalue distribution. There exists a triple point $(\tau, \lambda) = (1/8, 3/2)$.