今後の研究計画（英訳）

I will study the tails of knots and the skein algebras of surfaces via the linear skein theory associated with various Lie algebras. These studies are related to invariants of three-manifolds and mathematical physics by specializing the quantum parameter q to a root of unity. I expect that the studies on tails and skein algebras involve various topics in other fields.

■Study on the tail of a knot The existence of the tail of a knot is interpreted to the stability of the coefficients of the colored $\mathfrak{g}$ Jones polynomials of the knot. Garoufalidis–Lê (2015) showed that the colored $\mathfrak{sl}_2$ Jones polynomial of alternating knots have the k-th stability for any non-negative integer k. It implies the existence of k-th tail. We remark that the 0-th tail is just the tail of a knot. We will study the k-th tail for higher simple Lie algebras $\mathfrak{g}$ by giving explicit formulas of the colored $\mathfrak{g}$ Jones polynomials.

We have confirmed the stability of the k-th coefficient, a refined version of Garoufalidis–Lê (2015), by using a computer. Furthermore, we confirm a new phenomenon that the k-th tail, for $k > k_0$, becomes a palindromic q-polynomials for some k_0. We will study the existence of the k-th tail and such phenomenon. Furthermore, we expect that the q-holonomicity of quantum $\mathfrak{g}$ invariants shown by Garoufalidis-Lê (2005) is related to the stability.

We expect that we can obtain explicit formulas of (false) theta series and Andrews-Gordon type identities for $\mathfrak{g}$ via explicit computations of the tail of $(2, m)$-torus links.

■Study on skein algebras I am going to study the algebraic properties of skein algebras associated with higher Lie algebras, for example, simple Lie algebras $\mathfrak{g} = \mathfrak{sl}_3, \mathfrak{sp}_4, \mathfrak{g}_2$ of rank 2. The aim of this research is to reveal the relation among the skein algebra, the quantum cluster algebra, and quantum Teichmüller spaces. Furthermore, we will consider applications to the geometry of moduli spaces of flat G-connections of surfaces and 3-manifolds. These researches will be carried out in collaboration with Tsukasa Ishibashi and Shunsuke Kano.

In the previous work with Tsukasa Ishibashi, we obtained showed correspondences of skein algebras and quantum cluster algebras for $\mathfrak{sl}_3$ and $\mathfrak{sp}_4$. For the quantum cluster algebra $\mathscr{A}^q_{\mathfrak{g},\Sigma}$, the quantum upper cluster algebra $\mathscr{U}^q_{\mathfrak{g},\Sigma}$, the skein algebra $\mathscr{S}^q_{\mathfrak{g},\Sigma}$, and boundary-localizaiton $\mathscr{S}^q_{\mathfrak{g},\Sigma}[\partial^{-1}]$ of it, we had

$$\mathscr{S}^q_{\mathfrak{g},\Sigma}[\partial^{-1}] \subset \mathscr{A}^q_{\mathfrak{g},\Sigma} \subset \mathscr{U}^q_{\mathfrak{g},\Sigma} \subset \mathrm{Frac}\mathscr{S}^q_{\mathfrak{g},\Sigma}.$$

We will study whether the equality holds for $\mathfrak{sl}_3$ and $\mathfrak{sp}_4$. In the case of $\mathfrak{g}_2$ and $\mathfrak{sl}_n$, we first study the above embedding of skein algebra into cluster algebra. We will also study the positivity of "elevation-pereserving webs".

We also plan to study the quantum representation of the mapping class group of surfaces using skein algebras associated with higher Lie algebras. I expect that the quantum representation of the mapping class group will help us to discover new invariants of 3-manifolds.