

# The 9th Japan-China Geometry Conference

Date: December 23 – 27, 2024.

Venue: Sunport Hall Takamatsu, Small Hall 1  
(Takamatsu City, Kagawa 760-0019)

# Schedule

## December 23

- Chair: Hiroshi Tamaru

10:00–10:20 opening, Hitoshi Moriyoshi (Nagoya University) and Gang Tian (Peking University)

- Chair: Ryushi Goto

10:30–11:20 Gang Tian (Peking University),

Title: Asymptotics of thresholds and related problems

11:30–12:20 Akito Futaki (Tsinghua University)

Title: Cartan Geometry and Infinite-Dimensional Kempf-Ness Theory

– lunch –

- Chair: Yoshihiro Ohnita

14:40–15:30 Wenjiao Yan (Beijing Normal University)

Title: Complex structure and isoparametric foliation

– tea break –

- Chair: Li Haizhong

15:50–16:40 Guofang Wang (Universitat Freiburg)

Title: A higher order geometric mass

16:50–17:40 Daguang Chen (Tsinghua University)

Title: Eigenvalue estimates of the Laplacian under Dirichlet and Robin Boundary Conditions

## December 24

- Chair: Fu Jixiang

10:00–10:50 Hajime Ono (University of Tsukuba)

Title: The Einstein-harmonic equations and constant scalar curvature Kähler metrics

– tea break –

- Chair: Li Jiayu

11:20–12:10 Jianchun Chu (Peking University)

Title: On Kähler manifolds with non-negative mixed curvature

– lunch –

- Chair: Feng Huitao

14:40–15:30 Kota Hattori (Keio University)

Title: The energy of maps accompanying the collapsing of the K3 surface

– tea break –

- Chair: Takashi Shioya

15:50–16:40 Huichun Zhang (Sun Yat-sen University)

Title: Regularity of harmonic maps between singular spaces

16:50–17:40 Tongrui Wang (Shanghai Jiao Tong University)

Title: Minimal surfaces with low genus in lens spaces

## December 25

- Chair: Zhang Weiping

9:30–10:20 Xiangyu Zhou (Chinese Academy of Sciences)

Title: Recent progress on multiplier ideal sheaves

– tea break –

- Chair: Reiko Miyaoka

10:40–11:30 Zhenxiao Xie (Beihang University)

Title: Willmore surfaces in 4-dimensional conformal manifolds

11:40–12:30 Hisashi Kasuya (Osaka University)

Title: Non-abelian Hodge correspondence and moduli spaces on Sasakian manifolds

– lunch –

- Chair: Tang Zizhou

14:40–15:30 Guoxin Wei (South China Normal University)

Title: Complete  $\lambda$ -hypersurfaces in Euclidean spaces

– tea break –

- Chair: Hitoshi Moriyoshi

15:50–16:40 Masashi Yasumoto (Tokushima University)

Title: Discretization of timelike isothermic surfaces, and beyond

16:50–17:40 Guangxiang Su (Nankai University)

Title: Llarull's theorem on odd dimensional noncompact manifolds

## December 26

9:00–18:00 Free Discussion

## December 27

- Chair: Ding Qing

10:00–10:50 Xinan Ma (University of Science and Technology of China)

Title: Rigidity result for semilinear subelliptic  
partial differential equation on Cauchy-Riemannian manifold.

– tea break –

11:10–12:00 Jun Sun (Wuhan University)

Title: On the symplectic and Lagrangian mean curvature flows

– lunch –

- Chair: Ayato Mitsuishi

14:20–15:10 Tatsuya Miura (Kyoto University)

Title: Delta-convex structure of the singular set of distance functions

– tea break –

- Chair: Shouhei Honda

15:30–16:20 Xi Zhang (Nanjing University of Science and Technology)

Title: Rational connectedness and curvature positivity

16:30–17:20 Kazuo Akutagawa (Chuo University)

Title: Harmonic maps from the product of hyperbolic planes to hyperbolic spaces

- Chair: Hiroshi Tamaru

17:20–17:25 announcement for the 10th China-Japan Geometry Conference, Xi Zhang.

17:25–17:35 closing, Shouhei Honda (University of Tokyo) and Weiping Zhang (Nankai University)

# Abstract

**Gang Tian** (Peking University)

**Title:** Asymptotics of thresholds and related problems

**Abstract:** In this talk, I will discuss asymptotics of algebraic geometric invariants that are fundamental in the study of  $K$ -stability and canonical Kähler metrics, such invariants include the stability thresholds and global log canonical thresholds. I will also discuss their relations to discrete Okounkov bodies. If time permits, I will show how our study leads to a new higher-dimensional generalization of classical Weierstrass gap theory.

**Akito Futaki** (Tsinghua University)

**Title:** Cartan Geometry and Infinite-Dimensional Kempf-Ness Theory

**Abstract:** We pioneer the development of a rigorous infinite-dimensional framework for the Kempf-Ness theorem, addressing the significant challenge posed by the absence of a complexification for the symmetry group in infinite dimensions, e.g, the diffeomorphism group. We propose a novel approach, based on Cartan bundles, to generalize Kempf-Ness theory to infinite dimensions, invoking the fundamental role played by the Cartan connection. This approach allows us to define and study objects essential for the Kempf-Ness theorem, such as the complex model for orbits and the Kempf-Ness functional, as well as establishing its convexity properties and defining an obstruction to the existence of zeros of the moment map. We show how our framework can be applied to the study of various problems in Kähler geometry, deformation quantization, and gauge theory. This talk is based on a joint work with Tobias Diez and Tudor S. Ratiu, arXiv:2405.20864.

**Wenjiao Yan** (Beijing Normal University)

**Title:** Complex structure and isoparametric foliation

**Abstract:** In this talk, we will introduce our recent progress on the almost complex and complex structures on  $S^6$  and on certain isoparametric foliation in the unit sphere. This talk is based on joint works with Professor Zizhou Tang and Professor Chao Qian.

**Guofang Wang** (Universitat Freiburg)

**Title:** A higher order geometric mass

**Abstract:** In the talk we will first introduce a higher order mass by using a generalized scalar curvature and discuss its properties. It relates various geometric quantities and geometric inequalities. The talk bases on joint work with Yuxin Ge (Toulouse), Jie Wu (Zhejiang Uni.) and Wei Wei (Nanjiang Uni.)

**Daguang Chen** (Tsinghua University)

**Title:** Eigenvalue estimates of the Laplacian under Dirichlet and Robin Boundary Conditions

**Abstract:** In this talk, we will present the estimation of eigenvalues of the Laplacian operator under Dirichlet and Robin boundary conditions. We will begin by discussing universal inequalities associated with the Laplacian under Dirichlet boundary conditions and their connection to Li-Yau inequalities. We will also talk about the Bossel-Daners inequality of the Laplacian with Robin boundary on Riemannian manifolds. We will explain the inequality in detail and discuss their application. This talk is based on the joint work with Professor Qingming Cheng (Fukuoka University) and Professor Haizhong Li (Tsinghua University)

**Hajime Ono** (University of Tsukuba)

**Title:** The Einstein-harmonic equations and constant scalar curvature Kähler metrics

**Abstract:** The Einstein-Maxwell equations play an important role in general relativity. LeBrun pointed out that the Einstein-Maxwell equations on Kähler surfaces are interesting from geometric point of view. In this talk, we introduce a generalization of the Einstein-Maxwell equations, called the Einstein-harmonic equations. Then we prove that constant scalar curvature Kähler manifolds of even complex dimension give solutions of the Einstein-harmonic equations.

**Jianchun Chu** (Peking University)

**Title:** On Kähler manifolds with non-negative mixed curvature

**Abstract:** The mixed curvature comes from a linear combination of the Ricci and holomorphic sectional curvature. In this talk, we will first survey recent progress. Then we discuss a structure theorem for compact Kähler manifolds with non-negative mixed curvature. This is a joint work with Man-Chun Lee and Jintian Zhu.

**Kota Hattori** (Keio University)

**Title:** The energy of maps accompanying the collapsing of the K3 surface

**Abstract:** In this talk, I explain the Dirichlet energy of some smooth maps associated with a collapsing family of hyper-Kähler metrics on the K3 surface constructed by Foscolo. I introduce an invariant for homotopy classes of smooth maps, showing that it gives a lower bound of the energy. Moreover, when the hyper-Kähler metrics collapse to a 3-dimensional flat orbifold, the energy converges to the above invariant.

**Huichun Zhang** (Sun Yat-sen University)

**Title:** Regularity of harmonic maps between singular spaces

**Abstract:** In this talk, we introduce the development of regularity of harmonic maps from Alexandrov spaces/RCD-spaces into metric spaces with nonpositive curvature in the sense of Alexandrov, including the interior Lipschitz regularity and an optimal boundary regularity. This is based on the joint works with Xi-Ping Zhu.

**Tongrui Wang** (Shanghai Jiao Tong University)

**Title:** Minimal surfaces with low genus in lens spaces

**Abstract:** In this talk, I will discuss two either-or results for the multiple existences of minimal real projective planes and minimal Klein bottles in certain lens spaces with generic metrics. In particular, we show in positive Ricci  $\mathbb{R}P^3$  that there are four distinct minimal real projective planes together with four distinct minimal tori, and the number of minimal tori can be improved to five for almost all metrics of positive Ricci. Our proof is mainly based on a variant multiplicity one theorem for the Simon-Smith min-max theory under certain equivariant settings. This talk is based on the joint work with Xingzhe Li and Xuan Yao.

**Xiangyu Zhou** (Chinese Academy of Sciences)

**Title:** Recent progress on multiplier ideal sheaves

**Abstract:**

**Zhenxiao Xie** (Beihang University)

**Title:** Willmore surfaces in 4-dimensional conformal manifolds

**Abstract:** In this talk, we show the first and second variational formulas of the Willmore functional for closed surfaces in 4-dimensional conformal manifolds. As an application, the Clifford torus in  $\mathbb{C}P^2$  is proved to be strictly Willmore-stable. This provides a strong support to the Willmore-type conjecture of Montile and Urbano, which states that the Clifford torus in  $\mathbb{C}P^2$  minimizes the Willmore functional among all tori. In 4-dimensional locally symmetric spaces, by constructing some holomorphic differentials, we prove that among all minimal 2-spheres only those super-minimal ones can be Willmore. This is a joint work with Prof. Changping Wang.

**Hisashi Kasuya** (Osaka University)

**Title:** Non-abelian Hodge correspondence and moduli spaces on Sasakian manifolds

**Abstract:** Hitchin, Corlette and Simpson established the correspondence between flat bundles and Higgs bundles on compact Kähler manifolds. This correspondence is called the non-abelian Hodge correspondence. Sasakian geometry is an odd-dimensional analogy of Kähler geometry. In the joint works with I. Biswas, we investigate the non-abelian Hodge correspondence on compact Sasakian manifolds. In this talk, we give a detailed description of the non-abelian Hodge correspondence on a compact Sasakian manifold at the level of moduli spaces.

**Guoxin Wei** (South China Normal University)

**Title:** Complete  $\lambda$ -hypersurfaces in Euclidean spaces

**Abstract:**  $\lambda$ -hypersurfaces are stationary solutions to the isoperimetric problem in the Gaussian space, 0-hypersurfaces are exactly self-shrinkers of mean curvature flow. In this talk, we give a survey about examples, rigidity theorems and classification theorems of complete  $\lambda$ -hypersurfaces in Euclidean space. Especially, we focus on embedded examples of  $\lambda$ -hypersurfaces in Euclidean spaces.

**Masashi Yasumoto** (Tokushima University)

**Title:** Discretization of timelike isothermic surfaces, and beyond

**Abstract:** A new research field "discrete differential geometry" concerns discretizations of differential geometry that preserve as many smooth structures as possible. In particular, initiated by Bobenko, Pinkall, Suris, and so on, integrable discretizations of surfaces have achieved great success in this direction.

In the smooth case, timelike isothermic surfaces in Lorentz-Minkowski 3-space  $\mathbb{L}^3$  are parametrized by Lorentz conformal curvature line coordinates, and many important surface classes such as timelike minimal surfaces and timelike constant mean curvature surfaces are included partially. On the other hand, unlike the case of the Euclidean case, timelike minimal surfaces and timelike constant mean curvature surfaces are not necessarily timelike isothermic.

In this talk, we discuss a discretization of discrete timelike isothermic surfaces in  $\mathbb{L}^3$ . In particular, we focus on discrete timelike isothermic surfaces in the Lorentz-Minkowski 2-plane, equivalently discrete para-holomorphic functions in the para-complex plane. As an application, we derive a construction of discrete timelike isothermic minimal surfaces in  $\mathbb{L}^3$ , and describe discrete timelike minimal surfaces that are not necessarily discrete timelike isothermic. This is the first step toward a unified discretization of all the discrete timelike constant mean curvature surfaces in  $\mathbb{L}^3$ .

**Guangxiang Su** (Nankai University)

**Title:** Llarull's theorem on odd dimensional noncompact manifolds

**Abstract:** Let  $(M, g^{TM})$  be an odd dimensional ( $\dim M \geq 3$ ) connected oriented noncompact complete spin Riemannian manifold. Let  $k^{TM}$  be the associated scalar curvature. Let  $f : M \rightarrow S^{\dim M}(1)$  be a smooth area decreasing map which is locally constant near infinity and of nonzero degree. Suppose  $k^{TM} \geq (\dim M)(\dim M - 1)$  on the support of  $df$ , we show that  $\inf(k^{TM}) < 0$ . This is a joint work with Yihan Li, Xiangsheng Wang and Weiping Zhang.

**Xinan Ma** (University of Science and Technology of China)

**Title:** Rigidity result for semilinear subelliptic partial differential equation on Cauchy-Riemannian manifold.

**Abstract:** On CR manifolds we get the rigidity result, i.e., subelliptic equations have no other solution than some constant at least when parameters are in a certain range, thus solved also the conjecture of Xiaodong Wang in his Math. Z. 2022 paper, in Riemannian geometry version the corresponding result was got by Bidaut Veron- Veron in 1991. The rigidity result also deduces the best constant for the Folland-Stein Sobolev inequality on closed CR manifolds, when the CR manifold is  $S^{2n+1}$  this inequality was obtained by Frank-Lieb in 2012. This is a joint work with Qianzhong Ou and Tian Wu.

**Jun Sun** (Wuhan University)

**Title:** On the symplectic and Lagrangian mean curvature flows

**Abstract:** We will talk about our recent results on singularity analysis for the symplectic mean curvature flow and Lagrangian mean curvature flow, especially on translating solitons to such flows. This talk is based on joint work with Professor Xiaoli Han and Professor Jiayu Li.

**Tatsuya Miura** (Kyoto University)

**Title:** Delta-convex structure of the singular set of distance functions

**Abstract:** This talk is about the structure of the singular set of the distance function from an arbitrary closed subset of the standard Euclidean space, or more generally of a complete Finsler manifold. In terms of PDE, the distance function can be viewed as a viscosity solution to the classical eikonal equation or its generalization. Our main result, obtained jointly with Minoru Tanaka (Tokai University), shows that the singular set is equal to a countable union of delta-convex hypersurfaces up to an exceptional set of codimension two. This result is new even in the standard Euclidean space and optimal in view of regularity.

**Xi Zhang** (Nanjing University of Science and Technology)

**Title:** Rational connectedness and curvature positivity

**Abstract:** In this talk, we consider asymptotic behavior of the perturbed Hermitian-Yang-Mills equation, and establish the correspondence between rational connectedness in algebraic geometry and mean curvature positivity in differential geometry. We also introduce our recent work on the structure of compact Kähler manifolds with nonnegative holomorphic sectional curvature. These works are joint with Chao Li, Shiyu Zhang and Chuanjing Zhang.

**Kazuo Akutagawa** (Chuo University)

**Title:** Harmonic maps from the product of hyperbolic planes to hyperbolic spaces

**Abstract:** In this talk, we will consider the asymptotic Dirichlet problem for harmonic maps from the product  $\mathbb{H}^{m_1} \times \mathbb{H}^{m_2}$  of two hyperbolic spaces to hyperbolic spaces. It remarks that  $\mathbb{H}^{m_1} \times \mathbb{H}^{m_2}$  is a higher rank symmetric space of noncompact type. We first show uniqueness and non-existence results, particularly the existence of such harmonic maps (with some natural conditions) implies that it must be  $m_1 = m_2 = 2$ . We also show an existence result for harmonic maps from  $\mathbb{H}^2 \times \mathbb{H}^2$  to hyperbolic spaces. This is a joint work with Yoshihiko Matsumoto.

謝辞

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